NAG Toolbox for MATLAB

d01bb

1 Purpose

d01bb returns the weights and abscissae appropriate to a Gaussian quadrature formula with a specified number of abscissae. The formulae provided are Gauss–Legendre, Gauss–Rational, Gauss–Laguerre and Gauss–Hermite.

2 Syntax

```
[weight, abscis, ifail] = d01bb(d01xxx, a, b, itype, n)
```

3 Description

d01bb returns the weights and abscissae for use in the Gaussian quadrature of a function $f(x)$. The quadrature takes the form

$$S = \sum_{i=1}^{n} w_i f(x_i)$$

where w_i are the weights and x_i are the abscissae (see Davis and Rabinowitz 1975, Fröberg 1970, Ralston 1965 or Stroud and Secrest 1966).

Weights and abscissae are available for Gauss–Legendre, Gauss–Rational, Gauss–Laguerre and Gauss–Hermite quadrature, and for a selection of values of n (see Section 5).

(a) Gauss–Legendre Quadrature:

$$S \simeq \int_a^b f(x)\,dx$$

where a and b are finite and it will be exact for any function of the form

$$f(x) = \sum_{i=0}^{2n-1} c_i x^i.$$

(b) Gauss–Rational quadrature:

$$S \simeq \int_a^\infty f(x)\,dx \qquad (a+b>0) \qquad \text{or} \qquad S \simeq \int_{-\infty}^a f(x)\,dx \qquad (a+b<0)$$

and will be exact for any function of the form

$$f(x) = \sum_{i=2}^{2n+1} \frac{c_i}{(x+b)^i} = \frac{\sum_{i=0}^{2n-1} c_{2n+1-i}(x+b)^i}{(x+b)^{2n+1}}.$$

(c) Gauss–Laguerre quadrature, adjusted weights option:

$$S \simeq \int_a^\infty f(x)\,dx \qquad (b>0) \qquad \text{or} \qquad S \simeq \int_{-\infty}^a f(x)\,dx \qquad (b<0)$$

and will be exact for any function of the form

$$f(x) = e^{-bx} \sum_{i=0}^{2n-1} c_i x^i.$$

(d) Gauss–Hermite quadrature, adjusted weights option:

$$S \simeq \int_{-\infty}^{+\infty} f(x)\,dx$$

and will be exact for any function of the form

$$f(x) = e^{-b(x-a)^2} \sum_{i=0}^{2n-1} c_i x^i \qquad (b > 0).$$

(e) Gauss–Laguerre quadrature, normal weights option:

$$S \simeq \int_a^{\infty} e^{-bx} f(x)\,dx \qquad (b > 0) \qquad \text{or} \qquad S \simeq \int_{-\infty}^{a} e^{-bx} f(x)\,dx \qquad (b < 0)$$

and will be exact for any function of the form

$$f(x) = \sum_{i=0}^{2n-1} c_i x^i.$$

(f) Gauss–Hermite quadrature, normal weights option:

$$S \simeq \int_{-\infty}^{+\infty} e^{-b(x-a)^2} f(x)\,dx$$

and will be exact for any function of the form

$$f(x) = \sum_{i=0}^{2n-1} c_i x^i.$$

Note: that the Gauss–Legendre abscissae, with $a = -1$, $b = +1$, are the zeros of the Legendre polynomials; the Gauss–Laguerre abscissae, with $a = 0$, $b = 1$, are the zeros of the Laguerre polynomials; and the Gauss–Hermite abscissae, with $a = 0$, $b = 1$, are the zeros of the Hermite polynomials.

4 References

Davis P J and Rabinowitz P 1975 *Methods of Numerical Integration* Academic Press

Fröberg C E 1970 *Introduction to Numerical Analysis* Addison–Wesley

Ralston A 1965 *A First Course in Numerical Analysis* pp. 87–90 McGraw–Hill

Stroud A H and Secrest D 1966 *Gaussian Quadrature Formulas* Prentice–Hall

5 Parameters

5.1 Compulsory Input Parameters

1: **d01xxx – string**

String specifying the quadrature formula to be used:

`'d01baz'`, for Gauss–Legendre weights and abscissae;

`'d01bay'`, for Gauss–Rational weights and abscissae;

`'d01bax'`, for Gauss–Laguerre weights and abscissae;

`'d01baw'`, for Gauss–Hermite weights and abscissae.

2: **a – double scalar**

3: **b – double scalar**

The quantities a and b as described in the appropriate sub-section of Section 3.

4: **itype – int32 scalar**

Indicates the type of weights for Gauss–Laguerre or Gauss–Hermite quadrature (see Section 3).

itype $= 1$

Adjusted weights will be returned.

itype $= 0$

Normal weights will be returned.

Constraint: **itype** $= 0$ or 1.

For Gauss–Legendre or Gauss–Rational quadrature, this parameter is not used

5: **n – int32 scalar**

n, the number of weights and abscissae to be returned.

Constraint: **n** $= 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 48$ or 64.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **weight(n) – double array**

The **n** weights. For Gauss–Laguerre and Gauss–Hermite quadrature, these will be the adjusted weights if **itype** $= 1$, and the normal weights if **itype** $= 0$.

2: **abscis(n) – double array**

The **n** abscissae.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The N-point rule is not among those stored. If the soft fail option is used, the weights and abscissae returned will be those for the largest valid value of **n** less than the requested value, and the excess elements of **weight** and **abscis** (i.e., up to the requested **n**) will be filled with zeros.

ifail $= 2$

The value of **a** and/or **b** is invalid.

Gauss–Rational: $\mathbf{a} + \mathbf{b} = 0$
Gauss–Laguerre: $\mathbf{b} = 0$
Gauss–Hermite: $\mathbf{b} \leq 0$

If the soft fail option is used the weights and abscissae are returned as zero.

ifail = 3

Laguerre and Hermite normal weights only: underflow is occurring in evaluating one or more of the normal weights. If the soft fail option is used, the underflowing weights are returned as zero. A smaller value of **n** must be used; or adjusted weights should be used (**itype** = 1). In the latter case, take care that underflow does not occur when evaluating the integrand appropriate for adjusted weights.

7 Accuracy

The weights and abscissae are stored for standard values of **a** and **b** to full machine accuracy.

8 Further Comments

Timing is negligible.

9 Example

```
a = 0;
b = 1;
itype = int32(1);
n = int32(6);
[weight, abscis, ifail] = d01bb('d01bax', a, b, itype, n)
```

```
weight =
    0.5735
    1.3693
    2.2607
    3.3505
    4.8868
    7.8490
abscis =
    0.2228
    1.1889
    2.9927
    5.7751
    9.8375
   15.9829
ifail =
           0
```